Photon redshift and the appearance of a naked singularity

I. H. Dwivedi

Tata Institute of Fundamental Research
Homi Bhabha Road, Bombay 400 005
India

ABSTRACT

In this paper we analyze the redshift as observed by an external observer receiving photons which terminate in the past at the naked singularity formed in a Tolman-Bondi dust collapse. Within the context of models considered here it is shown that photons emitted from a weak curvature naked singularity are always finitely redshifted to an external observer. Certain cases of strong curvature naked singularities, including the self-similar one, where the photons are infinitely redshifted are also pointed out.

I. INTRODUCTION

It is generally believed that star with sufficient mass undergoing a gravitational collapse would release very high energy during the collapse process. For an external observer, gravitational collapse of a spherically symmetric star, appears to take an infinite time (observer time T_o) to reach its Schwarzschild radius $2M$. Theoretically therefore a collapsing star should be visible for ever. The redshift[1] z (frequency shift) in late stages of collapse, however,

$$1 + z \propto \exp{(\frac{t_o}{4M})} \tag{1}$$

increases exponentially with a very short characteristic time(and subsequently the luminosity decays exponentially with a short characteristic time) and hence a collapsing star would rather disappear in a short period of time. The end state of such a gravitational collapse could either be a naked singularity or a blackhole. The end result of gravitational collapse is rather crucial to cosmic censorship according to which a gravitational collapse must necessarily end in a singularity that is covered i.e. no light rays can escape to an external observer from the singularity thus formed. Recently many examples of naked singularities have been given[2−6]. We can categorize these naked singularity examples into basically two categories. In the first kind the naked singularity has only a single future directed radial null geodesic terminating at the singularity in the past while in second there is a family of radial null geodesics terminating at the naked singularity in the past. Therefore while the former kind of the naked singularity would only be visible for a instant the later one could be visible to an external observer for ever. The redshift factor therefore becomes very important because if the photons escaping from the naked singularity are infinitely redshifted as calculations for certain very few models so far suggest[3,4], it could be argued that these examples of naked singularities though theoretically admissible in general relativity would, however, not be visible to external observer.

The aim in this paper is therefore to analyze the redshift from this perspective. Tolman-Bondi dust collapse models have been investigated quite extensively in this context. It has been shown[3,6] that wide classes of solutions within the Tolman-Bondi spacetime do allow the formation of both strong curvature and weak curvature naked singularities of either categories as mentioned above. We investigate these dust collapse models for redshift analysis. We do find that for self-similar dust models, photons are infinitely redshifted

as shown in earlier work[4]. For the dust models considered here and where the naked singularity is a strong curvature singularity, the redshift is also infinite. However, for dust models where the singularity is a weak curvature singularity [9] suitably characterized by the mass function $F(r)$ and the energy function $f(r)$, the redshift is finite. Strong curvature naked singularities9 are thus physically censored in the sense that no amount of energy would escape from the singularity on account of infinite redshift, the weak curvature naked singularity on the other hand have a finite redshift and may be instantaneously visible. For globally naked weak curvature singularities the redshift, for photons terminating in the past at the naked singularity and crossing the boundary of the star at late stages near Schwarzschild radius to reach the distant observer, increases exponentially but with a different time constant compared to the photons emitted from the surface of the star. Thus it is likely that luminescence of the naked singularity and that of the surface of the star would not disappear simultaneously in such cases.

The redshift factor is defined as follows. Consider a source with 4-velocity $u^a_{(s)}$ and an observer with 4-velocity $u^a_{(o)}$ located at events P_1 and P_2 in an arbitrary spacetime. Let $k^a \equiv dx^a/d\lambda$ be the tangent vector to the null geodesic connecting the two events P_1 and P_2 with λ being the affine parameter. The redshift z (i.e. frequency shift) is defined as

$$1 + z = \frac{[k_a u^a_{(s)}]_{P_1}}{[k_a u^a_{(o)}]_{P_2}} \tag{2}$$

where the numerator and the denominator are evaluated at the events P_1 and P_2 respectively which denote emission and reception of light. In the above relation the source or the observer need not be in the same coordinate patch. Thus the source for example could be in the interior dust cloud described by the Tolman-Bondi solutions, while the observer could be a Schwarzschild observer in the external Schwarzschild spacetime. However, at the boundary the geodesics connecting the two events must be continuous.

In order to calculate the redshift, besides having the expressions for $u^a_{(s)}, u^a_{(o)}$ one must have the trajectory of photons as well as the tangent k^a to the null geodesic connecting the event P_1 at the source and P_2 at the observer and one must evaluate them as per equation (2) at the event P_1 when the light emitted and at P_2 when it is received. To do that we therefore discuss below Tolman-Bondi dust models and escaping photons from singularities in such a collapse.

II. Tolman-Bondi Dust collapse

Spherically symmetric dust collapse in the comoving coordinates is given by the Tolman-Bondi metric[7]

$$ds^2 = -dt^2 + \frac{R'^2}{1+f}dr^2 + R^2(d\theta^2 + sin^2\theta d\phi^2) \tag{3}$$

where $R = R(t,r)$ is interpreted as radius of the spherical shells in the sense that $4\pi R^2(t,r)$ gives the proper area of shells and is given by

$$\dot{R} = -\sqrt{\frac{F}{R} + f} \tag{4}$$

The dot and prime denote partial derivatives with respect to the coordinates t and r respectively. Arbitrary functions $F(r) \geq 0$ and $f(r)$ are atleast C^2 functions of coordinate r throughout the dust cloud and are interpreted as mass and energy functions respectively. The collapse leads to the formation of a shell focusing singularity at points described by the curve $t = t_o(r)$ such that $R(t_o(r), r) = 0$. The singularity at $r = 0$ is the central singularity. Only central singularity could be naked while singularities at $r > 0$ are necessarily covered. The singularity at $r = 0$ occurs at time $t = t_o(0) = t_s$ such that $R(t_s, 0) = 0$. Energy conditions are required to be satisfied throughout the spacetime and we would consider only the cases where no shell crossing singularities occur i.e. $R' > 0$ throughout the spacetime except perhaps at the central shell focusing singularity. Equation (4) can be integrated with the condition that at some initial $t = 0$, $R(0,r) = r$ to yield the expression for $R(t,r)$. Since in this paper mostly we would be interested in only marginally bound cases characterized by $f(r) = 0$ we would give the $R = R(t,r)$ for marginally bound case below and refer the reader for general expression of $R(t,r)$ to reference [6]

$$\frac{2}{3}\sqrt{\frac{R^3}{F}} = t_o(r) - t, \quad t_o(r) = \frac{2r^{\frac{3}{2}}}{3\sqrt{F}} \tag{5}$$

In the context of cosmic censorship the spacetime at some initial should be singularity free and hence we can take the following form of the mass function as a representative of a more general form (except in self-similar case) as

$$F(r) = F_o r^3(1 - F_1 r^\beta) \equiv F_o r^3 h(r) \tag{6}$$

where F_0, F_1 and $\beta \geq 1$ are positive constants. Note that the form $F(r) = F_o r^3 h(r)$ where $h(r)$ is at least a C^1 function of r is due to the requirement that at the initial $t = 0$ spacetime be nonsingular. First spacetime singularity for mass function given as above occurs at the center $r = 0$ at time $t = t_s = \frac{2}{3\sqrt{F_o}}$. The form of $F(r)$ in (6) is quite general except as mentioned in self-similar case where necessarily

$$F = F_o r \tag{7}$$

and in such a case the singularity occurs at $t = 0, r = 0$ while the spacetime is nonsingular for $t < 0$.

III. Photons from naked singularity

As pointed out in equation (2) the redshift factor is calculated from source to observer along the null geodesic(photon) connecting the source and the observer, hence the trajectory of light connecting the source and the observer has to be known. In present context it amounts to null geodesics escaping from the central naked singularity and reaching the external observer which could be either a local observer or a distant one. Tolman-Bondi dust collapse has been extensively studied in this context of formation of naked singularities and the escaping null geodesics from it[3,6]. It has been shown that for wide classes of dust models characterized by the arbitrary functions f and F the central singularity would be naked. Therefore avoiding the repetition of earlier work we would in this section only give essential expressions and results in this context and refer the reader to reference [6] for details.

Let k^a be tangent to radial null geodesics (i.e. $k^a k_a = 0 = k^a_{;b} k^b$) For the metric in equation (3) we have for radial null geodesics

$$k^r = \frac{\sqrt{1+f}}{R'} k^t \Rightarrow \frac{dt}{dr} = \frac{R'}{\sqrt{1+f}} \tag{8}$$

$$\frac{dk^t}{d\lambda} = -\frac{\dot{R}'}{\sqrt{1+f}} k^t k^r \tag{9}$$

where λ is an affine parameter. First of the above equation on integration gives the trajectories of photons as $t = t(r)$. While the later one can be integrated to obtain the expression for k^t. Restricting ourself from now on to marginally bound case $f = 0$ given by equation (5) and following reference [6] we use R and $u = r^\alpha$ as pair of variables and use (5) to write the radial

null geodesic equation in (8) as

$$\frac{dR}{du} = \frac{1}{\alpha}(1 - \sqrt{\frac{\Lambda}{X}})H = U(X,u) \tag{10}$$

$$\Lambda = \frac{F(r)}{r^{\alpha}}, \quad X = \frac{R}{r^{\alpha}}, H = H(X,r) = \frac{\eta}{3}X + \frac{\Theta}{\sqrt{X}} \tag{11}$$

where

$$\alpha = 1 + \frac{2\beta}{3}, \quad \Theta = \frac{\beta F_1}{3(1 - F_1 r^{\beta})}, \quad \eta = 3(1 - \Theta r^{\beta}) \tag{12}$$

for the mass function given in equation (6) and

$$\alpha = \eta = 1, \quad \Theta = \frac{2}{3} \tag{13}$$

for self-similar case given by mass function in equation (7). The integration of equation (10) gives the equation of radial null geodesics in the form $R(t,r) = y(r)$ where $y(r)$ is some function of r. The equation (10) is highly nonlinear and an exact solution even in very simple cases is not available except for self-similar case[4,6]. We would therefore analyze the behavior of radial null geodesics which is crucial to any calculation of redshift factor. To find this behavior we follow the earlier work and give the necessary discussion of outgoing radial null geodesics from the naked singularity below and would refer the reader to reference [6] for details.

The central singularity is at least locally naked if equation $V(X) \equiv U(X,0) - X = 0$ has a real positive root $X = X_0$

$$V(X) = U(X,0) - X = \left(1 - \sqrt{\frac{\Lambda_0}{X}}\right)(\frac{\eta_0}{3}X + \frac{\Theta_0}{\sqrt{X}}) - \alpha X = 0 \tag{14}$$

where $\eta_0 = \eta(0), \Theta_0 = \Theta(0)$. In case a real positive root $X = X_o$ of the root equation (14) exists then there are out going radial null geodesics from the naked singularity and the behavior of the geodesics in the neighborhood of the singularity is described by

$$R = X_0 u \tag{15}$$

Using equations (8) (10) the behavior in (15) of singular geodesic can be expressed in terms of t and r as

$$\frac{t}{t_s} = 1 + \frac{X_o r^\alpha}{1 - \sqrt{\frac{\Lambda_o}{X_o}}} \tag{16}$$

Note that for the general mass function given by equation (6) with $\beta < 3$, $V(X) = 0$ has a real positive root and is given by

$$V(X) = (1-\alpha)X + \frac{\Theta_0}{\sqrt{X}} = 0 \Rightarrow X_o^{\frac{3}{2}} = \frac{\Theta_0}{\alpha - 1} \tag{17}$$

For $\beta = 3$ case and self similar case given by equation (7) the root equation (14) turns out to be a biquadratic equation

$$V(X) \Longrightarrow 2(\sqrt{X})^4 + (\sqrt{X})^3\sqrt{\Lambda_0} - \Theta_0\sqrt{X} + \Theta_0\sqrt{\Lambda_0} = 0 \tag{18}$$

and have real positive roots if[6]

$$\frac{F_1}{F_o^{\frac{3}{2}}} > 13 + \frac{15}{2}\sqrt{3} \quad for \quad \beta = 3 \tag{19}$$

$$\frac{2}{F_o^{\frac{3}{2}}} > 13 + \frac{15}{2}\sqrt{3} \quad for \quad selfsimilar \tag{20}$$

respectively. Thus for the marginally bound cases considered here the singularity is naked and the behavior of outgoing radial null geodesics is given by (15) in the neighborhood of the singularity. For a external observer viewing the collapse, the singularity would be visible if null geodesics with their past end point at the singularity, reach the external observer within the the dust cloud or cross the boundary $r = r_b$ of the star with $dR/du > 0$. For this purpose we consider the paths of these null geodesics, which escape from the naked singularity with the tangent $X = X_o$(in (R, u) plane) which is the real positive root of Equation (14). By putting $x = X^{3/2}$, $x_0 = X_0^{3/2}$, $v(x) = \frac{3}{2}\sqrt{X}V(X)$ and $\mathcal{U}(x, u) = \frac{3}{2}\sqrt{X}U(X, u)$ we have after using equations (10), (11) and (14)

$$\frac{dx}{du} = \frac{\mathcal{U}(x, u) - \mathcal{U}(x, 0) + v(x)}{u} = \frac{(x - x_0)(h_0 - 1) + S}{u} \tag{21}$$

where we have put $v(x) = (x - x_0)(h_0 - 1) + h(x)$ such that $h_0 - 1 = (dv(x)/dx)_{x=x_0}$, $h(x)$ contains higher order terms in $(x - x_o)$ and $S = S(x,u) = \mathcal{U}(x,u) - \mathcal{U}(x,0) + h(x)$ and $S(x_o, 0) = 0$. On integration of above , the null trajectories $x = x(u)$ are given by,

$$x - x_0 = Du^{h_0-1} + u^{h_0-1}\int Su^{-h_0}du \tag{22}$$

Here D is a constant of integration that labels different geodesics[6]. Note that the last term in equation (22) always vanishes due to the fact that as $u \to 0$, $x \to x_0$ regardless of the value of the constant h_0. The first term on the right hand side of the equation (22) Du^{h_0-1}, however, vanishes only if $h_0 > 1$. Therefore, if $h_0 > 1$ a family of outgoing singular geodesics (with each curve being labeled by different values of the constant D) escape from the naked singularity with $X = X_0$ as the value of the tangent at the singularity in (R, u) plane. On the other hand, in case $h_0 < 1$, only single null geodesic with $D = 0$ escapes from the naked singularity with tangent $X = X_0$ in (R, u) plane. For $\beta = 3$ and the self-similar case[3,6] $h_0 > 1$ and hence a family of outgoing radial null geodesics escape from the singularity with each geodesics labeled by different value of D.

For the marginally bound cases considered here $h_0 = 1 - (\beta/\alpha) < 1$ for $\beta < 3$ and therefore only a single geodesic escapes from the naked singularity in this case given by $D = 0$. Furthermore $S(x,u) \to 0$ as $u \to 0$ for $\beta < 3$. An important point to note in this case is that the geodesics labeled by values of $D \neq 0$ represent radial null geodesics emitted from the center $r = 0$ with emission time $t = t_{s0} < t_s$. In fact from equations (5), (6) and (22) it follows as $r \to 0$ $D = [(R/r)^{3/2}]_{r=0} = 1 - (t_{s0}/t_s)$. D vanishes for the photon escaping from the naked central singularity at emission time $t_{s0} = t_s$ in such cases.

Thus radial null geodesics in the near regions of the naked central singularity can be described by.

$$R^{\frac{3}{2}} - X_o^{\frac{3}{2}}u^{\frac{3}{2}} = Du^{\frac{1}{2}+h_o} + u^{\frac{3}{2}}O(u^m) \quad m > 0 \tag{23}$$

where $O(u^m) \to 0$ as $u \to 0$.

The naked central singularity would be globally naked only if these out going radial null geodesics from the naked singularity would cross the boundary of the star before it reaches its Schwarzschild radius. In many examples

including the marginally bound cases the naked singularity is globally naked as well[2–6]. Therefore in cases when there are a family of null geodesics escaping from naked singularity which is globally naked as well, constant D in equations (22) is determined by the condition that at the boundary $u = u_b = r_b^\alpha$ and $x = (R_b/r_b^\alpha)^{3/2} = x_b$ and we have

$$x - x_0 = (x_b - x_0)\left[\frac{u}{u_b}\right]^{h_0 - 1} + u^{h_0 - 1}\int_{u_b}^{u} S u^{-h_0} du \tag{24}$$

The event horizon is represented by the null geodesic for which $x_b = \Lambda_b = (\Lambda(r_b))^{3/2}$. The null geodesics with x_b close to Λ_b cross the boundary of the star at late stages of collapse when the star radius is close to $2M$ (Schwarzschild radius). In case of a globally naked singularity where only single radial null geodesics escapes from the singularity, the escaping geodesic can cross the boundary at any stage of the collapse depending on the exact global behavior of function $F(r)$ and the boundary of the star at $r = r_b$. We should mention that our above discussion was limited only for marginally bound case, however, all the above qualitative description of outgoing radial null geodesics remain the same even for the general cases for $f \neq 0$ and the difference comes only on the value of α and in the form of equation $V(X) = 0$ in (14).

IV. Redshift

The aim in this paper is to evaluate redshift for light terminating at the central naked singularity in the past and received by an external observer. For the spacetime described by the Tolman-Bondi metric given in equation (3) we treat the source at $r = 0$ and the observer at $r = r_o$ within the dust cloud, Thus 4-velocities, $u^a_{(s)}$ of the source at the center $r = 0$ and $u^a_{(o)}$ of the observer at $r = r_o$ within the dust cloud are

$$u^a_{(s)} = \delta^a_t \tag{25}$$

$$u^a_{(o)} = \delta^a_t \tag{26}$$

Naked central singularity at $r = 0, t = t_s$ is the source of photons and is the event P_1 at the source. The radial null geodesic along which the redshift is to be evaluated are precisely the outgoing null geodesics which terminate at the singularity in the past and reach the external observer at event P_2. The expression k^t is important from the point of calculation for redshift and using

equations (5) to (11) we get by integrating equation (9)

$$k^t = c_o \exp\,(-\int \frac{\dot{R}'}{\sqrt{1+f}} k^r d\lambda) \Rightarrow = c_o \exp\,(-\int N(R,r)dr) \qquad (27)$$

where c_o is a constant and

$$N \equiv N(R,r) = \frac{\sqrt{rF}}{2R^2}\left(1 - \frac{\eta}{3} - \frac{2\eta}{3}(\frac{R}{r})^{3/2}\right) \qquad (28)$$

Using equations (25) to (28) and (2) we get the redshift factor for photons

$$1+z = \exp\,(\int_0^{r_o} N dr) \qquad (29)$$

For the general mass function given by equation (6) we can express

$$N = \frac{\sqrt{F_0 h(r)}}{2r^{\frac{\beta}{3}} X^2}\left(2(1-\Theta r^{\beta})X^{3/2} - \Theta\right) \qquad (30)$$

where $X = X(r)$ is to be treated as a function of r describing the path of null geodesics given by equation (22) or alternatively from equation (23). The lower limit $r = 0$ of the integral in equation (29) corresponds to the naked central singularity and is also a singular point of the integral. In order to determine the convergence of the integral let us consider the limit of

$$\lim_{r\to 0}[r^{\frac{\beta}{3}} N(X,r)] \qquad (31)$$

As per the discussion in the previous section on the radial null geodesics terminating at the naked central singularity at $t = t_s, r = 0$ the behavior of geodesics is given by equations (15) in the neighborhood of the singularity and in the limit of approach to singularity as $r \to 0, t \to t_s$ $X \to X_0$, hence using equations (10) to (12), (15) and (30) we get

$$\Psi = \lim_{r\to 0}[r^{\frac{\beta}{3}} N(X,r)] = \frac{\sqrt{F_o}}{2\sqrt{X_o}}(3-\alpha) \neq 0 \quad for \quad \beta < 3 \qquad (32)$$

$$\Psi = \lim_{r\to 0}[r^{\frac{\beta}{3}} N(X,r)] = \sqrt{\Lambda_0}(X_o + \frac{\Theta_0}{\sqrt{X_0}}) > 0 \quad for \quad \beta = 3 \qquad (33)$$

Since Ψ has a finite limiting value at the singularity for $\beta < 3$ the integral converges absolutely for $\beta < 3$ and therefore the redshift given in equation (29) for such a case is finite. On the other hand for $\beta = 3$ the limiting value is finite positive hence the integral in equation (29) diverges. A similar consideration in case of a selfsimilar spacetime given by the mass function in equation (7) reveals

$$\Psi = \lim_{r \to 0} [rN(X, r)] = \sqrt{\Lambda_0}(X_o + \frac{2}{3\sqrt{X_0}}) > 0 \tag{34}$$

and hence the integral diverges.

It would not be out of context to consider redshift from the point of view of an external observer viewing the photons emitted from the source at center at $r = 0$ approaching the naked central singularity at $r = 0$. The integral in equation (29) is too complicated to evaluate exactly however we can estimate the redshift factor and it's behavior and for the purpose of simplicity we can take the case $\beta = 1$ which corresponds to finite redshift as shown above. Equation (23) with constant $D = 1 - \frac{t_{so}}{t_s}$ in such a case describes the null geodesics from the center emitted at time $t = t_{so}$. Considering a local external observer at $r_o << 1$, using equations (6), (12), (23), (29) and (30) and neglecting higher order terms in r, we get for the redshift

$$1 + z \propto \exp(-3D^{\frac{2}{3}} + \frac{X_0^{\frac{3}{2}} r_o + 3D}{(X_0^{\frac{3}{2}} r_o + D)^{\frac{1}{3}}}) \tag{35}$$

As $t_{s0} \to t_s$, $D = 1 - \frac{t_{s0}}{t_s} \to 0$ and the redshift remains finite. Thus to an external observer viewing the source at the center $r = 0$ the redshift remains finite as the source at the center approaches the singularity at the center at time $t_{s0} = t_s$.

Thus we arrive at the conclusion that redshift is finite for collapse scenarios with $\beta < 3$ and diverges for $\beta = 3$ and selfsimilar cases. The interesting point to note that while $\beta < 3$ cases correspond to a weak curvature naked singularity the $\beta = 3$ and selfsimilar cases represent strong curvature singularities[6]. The conclusions would be the same in non marginally bound cases of dust collapse. Therefore while the strong curvature naked singularities would be physically censored owing to the infinite redshift the weaker curvature singularities can have finite redshift factor. This perhaps is the result of growth of density from outer layers of star to the center of the star

as characterized by the value of β. Thus if the density decreases away from the center faster as characterized by $\beta = 3$ the singularity is not only a strong curvature singularity but it also does not allow any energy to escape on the otherhand if the decrease in the density from the center is slow i.e. $\beta < 3$ the singularity is gravitationally weak and it allows the energy to escape.

V. Schwarzschild Observer

For a distant observer in external Schwarzschild spacetime viewing the late stages of a collapsing star, the redshift for the photons emitted from star,s surface appears to be time dependent and increasing exponentially. Since In the case of a globally naked singularity photons escaping from the central naked singularity can cross the boundary of the star closer and closer to the Schwarzschild radius. As a matter of interest we point out very briefly as a remark on the photon redshift for such a scenario. In the context of Tolman-Bondi dust collapse such a scenario is possible only for examples of globally naked central singularity where a family of null geodesics terminate at the singularity in the past and the singularity is gravitationally weak as well in view of the preceding section. it is quite likely that there are dust models in general where the singularity is not only weak but a family of null geodesics also terminates at the naked singularity[6,8]. In such cases after first ray crosses the boundary of the star in to vacuum there would always be geodesics escaping from the singularity to the Schwarzschild exterior closer and closer to the apparent horizon $R = F$ in the interior and would cross the boundary of the star closer and closer to the Schwarzschild radius $2M$. Let us consider a collapsing star with internal metric described by the Tolman-Bondi dust models and the exterior of the star is described by the vacuum Schwarzschild metric.

$$ds^2 = -(1 - \frac{2M}{r_s})dT^2 + \frac{dr_s^2}{1 - \frac{2M}{r_s}} + r_s^2(d\theta^2 + sin^2\theta d\phi^2) \tag{36}$$

Here r_s and $2M$ denote Schwarzschild radial coordinate and mass respectively. The junction conditions at the boundary $r = r_b$ are satisfied if $r_s = R_0(T) = R(t, r_b), F(r_b) = 2M$. The tangent vector to the geodesics in the interior is given by equations (8) and (9) for radial null geodesics. The tangent vector to radial null geodesics in the Schwarzschild exterior are

$$K^T = \frac{k}{1 - \frac{2M_s}{r}}, \quad K^r = k \tag{37}$$

where k is constant and is determined by the continuity of the tangent vectors to null geodesics in the interior and the exterior at the boundary

$$k = \frac{1 - \frac{F(r_b)}{R(t,r_b)}}{\sqrt{1+f(r_b)} + \sqrt{\frac{f(r_b)+F(r_b)}{R(t,r_b)}}}[k^t]_{r=r_b} \tag{38}$$

Using equations (36) to (38) we get for the redshift as observed by the distant observer at $r_s = r_1$ with four velocity $U^a_o = (\delta^a_T/\sqrt{1-\frac{2M}{r_1}})$ in the Schwarzschild exterior receiving photons which terminate in the past at the naked central singularity as

$$1 + z = \frac{(1-\frac{2M}{r_1})^{1/2}}{1-\sqrt{\frac{2M}{R_o}}} I \tag{39}$$

$$I = \exp\left(\int_0^{r_b} N dr\right) = \exp\left(\int_0^{R_b} N \frac{dr}{dR} dR\right) = \exp\left(\int_0^{R_b} \frac{dR}{R - F(r)} q(R,r)\right) \tag{40}$$

where we have put $f = 0$ and

$$q(R,r) = \sqrt{\frac{\Lambda}{X}}\left(1 + \sqrt{\frac{\Lambda}{X}}\right)\frac{\Theta - 2(1-\Theta r^\beta)X^{\frac{3}{2}}}{\Theta + (1-\Theta r^\beta)X^{\frac{3}{2}}} \tag{41}$$

value of q between the interval $(0, R_c)$ is finite and our interest is only in cases when R_b is close to $F(r_b) = 2M$ the main contribution to the integral I comes from the factor $R - F$ in the denominator, and the contribution of this term at the upper limit R_b very close to $F(r_b)$ is most significant. We therefore have after careful consideration the redshift behavior as

$$1 + z \propto \left(1 - \frac{2M}{R_o}\right)^{n-1} \propto e^{\frac{T_o(1-n)}{4m}} \tag{42}$$

where constant n depends on the $r_b, F(r_b)$ and we have used geodesic equations[1] (37) in Schwarzschild spacetime to express the result in terms of the observer time T_o. The point to note is that even the radiations from naked singularity in the late stages of collapse are redshifted which increase exponentially with a different characteristic time constant compared with the characteristic time for radiations coming from the surface of the star given in (1). Therefore the

radiations from the singularity would not necessarily disappear simultaneously with the radiations from the surface of the star in such cases.

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9. The weak singularity here means in the sense of Tipler, but they are strong in the sense of Krolak. It should however be noted that the singularities satisfying the Krolak condition also may not be extended through. Furthermore, a recent study (Deshingkar S., Dwivedi I. H. and Joshi P. S.; to appear) has shown that even the so called weak singularities in dust collapse are strong in the sense of Tipler as well, although they have a directional behavior as far as the strength of the singularity is concerned. Therefore, the strong sense here should be taken more in terms of the growth of density from outer layers towards the center.